

New contribution for the ballistic theory of the "variable stars".

Explanation of the phenomenon for the stars of the type U Geminorum and of the cluster type. by *M. La Rosa.*

1. The objections of Mr. *Bernheimer*¹⁾ and Mr. *Salet*²⁾ against my theoretical sketch of the phenomenon of the "variability" won from the application of the ballistic principle on the reproduction of the light caused me to a detailed investigation of the peculiarity, which let this sketch foresee us for the case that the constant becomes much larger kb of the principal equation than 1.

The results of such an investigation were very satisfying, because they led me to a simple and direct explanation of the behavior of the two groups of "variable stars", which had remained wrapped into a close secret until today.

The importance to stress such an agreement is redundant. It is a new group of facts, which naturally and directly favor the application of the ballistic principle to the light speak, an application, which presents itself on the basis of the quantum-theories as something easy and natural.

By the discussion of the divergence between the variability type, which is foreseen by the ballistic theory, and which, which (after *Bernheimer*) to have been observed is, for a later report reserves itself, wanting I only the analysis to state here, which led me to the announced results.

I will proceed from the investigation over the insignificant conditions of the equation:

$$\chi + a \cos \chi = K$$

on which the investigation runs out over the positions of a star, from which the emitted light arrives simultaneous with the observer, rotating in its course; an investigation, which has interest because of, which she can apply, to and in itself. Subsequently, I will state the method, which one with computation of the light curves in the most general possible case (any a , i.e. the overlay of the light coming from any number of positions) to obey can do, and their application in a concrete case ($a = 10\pi$) to accomplish, which will give me opportunity to prove the perfect analogy in the behavior between the light curves which can be foreseen for large a and the light curves which the observation resulted in for a long time for the variable ones of the type U Geminorum and the variable ones of a special character (cluster type), found in the star clusters.

2. It is reminded of the fact that the principal equation of my sketch of the ballistic theory of the variable stars³⁾

$$T = k\tau_0 + t + kb\tau_0 \cos \omega t$$

is, which during introduction of appropriate variability those form

$$y = x + a \cos x \tag{1}$$

or also

$$\theta = u - e \sin u \tag{1'}$$

assumes.

In this second form it is well well-known the astronomers, however only in the case were examined, in which $e < 1$ is, because it represents then the extremely simple connection, which with the movement of the planets between the time θ , which eccentric anomaly u and the eccentricity e of the path exist.

It is reminded further of the fact that the equation (1) graphically — in orthogonal Cartesian coordinates — to a "inclined Sinusoid" (see Fig. 1), one draws in

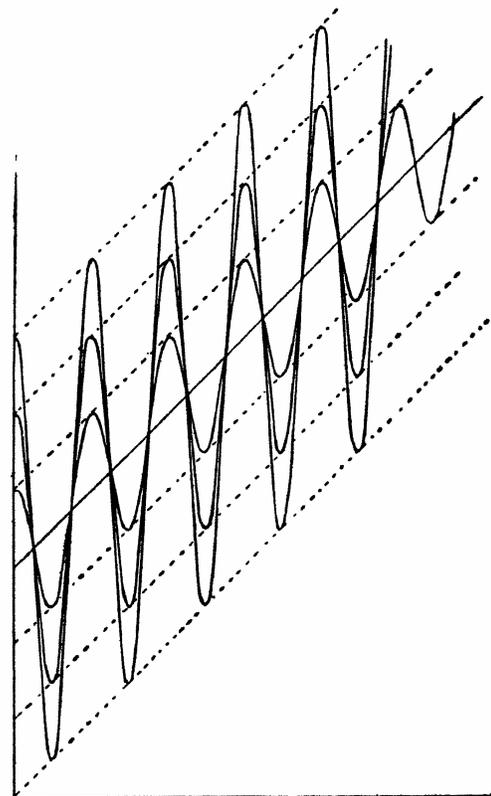


Fig. 1

¹⁾ Z. f. Phys. **36**.302, 1926. ²⁾ C. R. **183**.1263, 1926.

³⁾ the indications have the following meaning: t = time of departure of the light of the star rotating in its path, T = arrival time with the observer, τ_0 = rotating time of the star (in circular path), $\omega = 2\pi/\tau_0$ = angular speed of the star, b = relationship of the tangential speed v of the star to the normal speed of light c , $k = \Delta/c\tau_0$, where Δ the distance of the star to the observer. The case of the elliptical path was treated by *C. Cannata* in R. Acc. Lincei.

within the strip that the straight lines

$$y = x + a \quad y = x - a \quad (2)$$

to edges has. These are in infinitely many tangent points the upper in the points of abscissa to the curve:

$$x = 2n \pi$$

the lower in the points:

$$x = (2n + 1)\pi$$

with n as any whole number.

If one sets then in (1) $x \pm 2n \pi$ with whole n to the place of x , then one gets:

$$y(x + 2n \pi) = \pm 2n \pi + y(x). \quad (3)$$

This equation means us that with constant amplitude intervals of 2π the curve reproduces itself identically, whereby the only following change takes place: the new elbow proves regarding the preceding translation parallel around one to the y -axis shifted the constant amount of 2π .

This circumstance permits us to serve us of the terminology common for the repetitive functions by we as a period the curve section contained within an amplitude interval of 2π , as "amplitude" the factor a of the periodic member, as axis of the curve the straight line $y = x$, which is the axis of the strip, as phase in the point P the difference ¹⁾

$$x - 2n \pi$$

designate, where x the abscissa of P and n the ordinal number of the period is, to which the point P belonged, under which acceptance that one gives that period, which begins with $x = 0$ the index zero. With a word, we can treat the equation according to many regards (1), as if it would be a periodic ²⁾. One $a < 1$ accepts, always is the equation (1) so long growing, since it is not possible, the condition

$$\dot{y} = 1 - a \sin x = 0$$

to be sufficient, while the second differential \ddot{y} always > 0 remains; as soon as against it $a > 1$ is, it shows infinitely many maxima and infinitely many minima. From these first have the coordinates:

$$x_{M,n} = \alpha + 2n \pi \quad y_{M,n} = 2n \pi + (\alpha + a \cos \alpha)$$

the others:

$$x_{m,n} = (2n + 1)\pi - \alpha \quad y_{m,n} = (2n + 1)\pi - (\alpha + a \cos \alpha)$$

where n is any whole number (zero included) designation and α is the smallest value of $x > 0$, the equation (3) been sufficient; indeed

$$\alpha = \arcsin(1/a) \quad (4')$$

since $0 < \alpha < \frac{1}{2}\pi$.

This ahead-skillfully, we start to examine which is the number of the points, in which a straight line of the equation $y = K$ cuts the curve, or more exactly, to examine for a given value from a to how this number changes when varying K .

Owing to the "improper periodicity" of y , emphasized above, it is clear that it is sufficient, the investigation

to limit to only one period to accept i.e. K as variable an amplitude interval (2π) since for one period the found results can be expanded on the other periods. We want to assume K variability as between a and $a + 2\pi$. We emphasize first that the edges of the strip on the straight line $y = K$ separate a segment from constant length, whose ends have the abscissas $K - a$, $K + a$ and the fact that the looked for points — which are the roots of the equation $x + a \cos x = K$ — all on this segment to be must.

We begin with the acceptance that $K = a$, i.e. that the straight line cuts the curve at the first contact — index 0 — with the upper edge. If n is the number of the minima, which lie left from the end of $2a$ of our segment, then we can state that the number of the looked for intersections is $2n + 1$, since each minimum leads to two of such points, without first mentioned contact above to count, left of the first maximum lies and therefore to the right of the y axis of preceding minimum belongs directly.

The accepted condition that the index minimum $(n - 1)$ the latter is, which falls left from $2a$, leads to the other one that the ordinate of this minimum is the latter, which is $\leq a$. This becomes the inequality

$$y_{m,n-1} = (2n - 1)\pi - (\alpha + a \cos \alpha) \leq a$$

or

$$(2n - 1)\pi \leq a + y_{M,1}.$$

In order to make for us an exact conception of, what with the number of roots happens, if we let K grow, on the basis of a , constantly, is indicated it to hold itself the distances before eyes which have the first maximum and the index minimum n of the straight line $y = a$.

These distances are

$$\delta = y_{M,1} - a = \alpha + a(\cos \alpha - 1)$$

and

$$\eta = y_{m,n} - a = (2n + 1)\pi - (y_{M,1} + a).$$

It is easily apparent the fact that we in the case, where $\delta < \eta$ would be, any change in the number of the roots $2n + 1$ when growing K of a on $a + \delta = y_{M,1}$ (with inclusion of the extremes) to expect cannot, because while the straight line always cuts the comb of the first maximum with its gradual rising, does not succeed it to her yet to affect the index minimum n . On the other hand we must expect a reduction of this number around 2 units, if K , $y_{M,1}$ exceeding, which between this value and that values $y_{m,n}$ contained interval goes through.

Yes, there is then an amplitude interval

$$\eta - \delta = y_{m,n} - y_{M,1} = (2n + 1)\pi - 2y_{M,1} > 0$$

within it's the number of the roots on $2n - 1$ goes down, in order to return to $2n + 1$, as soon as K becomes equal to $y_{m,n}$. This value receives itself through the whole remainder of the period, i.e. in the interval $(y_{m,n}, a + 2\pi)$, invariably, there within the segment $K - a$, $K + a$ no loss at maxima still another profit at minima more to be received can.

¹⁾ So that the imported terminology for the sinusoidal functions of the common is similar, one would have to call the relationship $(x - 2n \pi)/2\pi$ as phase and angle value of the phase our difference. we will say

²⁾ For this reason that the equation (1) is an "improper periodic" function.

Briefly, if $(2n-1)$ is the largest odd whole number which meets the condition

$$(2n-1)\pi \leq a + y_{M,1}$$

and if we have besides

$$(2n+1)\pi > 2y_{M,1}$$

or at all

$$(2n-1)\pi \leq a + y_{M,1} < 2y_{M,1} < (2n+1)\pi$$

the looked for number of the roots varies in such a way between $(2n+1)$ and $(2n-1)$. It is $(2n+1)$ in the interval $(a, a + \delta)$, $(2n-1)$ in the interval $(a + \delta, a + \eta)$ and becomes again $(2n+1)$ in $(a + \eta, a + \omega)$. In the reason we have two intervals in each period, in whose the number of the roots $(2n+1)$ is, and which extends around $(2x - \eta)$ left from the contact with the top margin and around δ right from the same.

In the special is in the case, where

$$(2n-1)\pi = a + y_{M,1}$$

i.e. $y_{m,n} = a$, the number of the roots is $(2n+1)$ in the interval of δ and becomes $(2n-1)$ in all remaining parts, because the new minimum will be affected only at the end of the period.

Remains the trap examining, where $\delta \geq \eta$.

In first the same it happens that the straight lines $y = K$ at the same time tangent to the curve in that 1. maximum ($K = y_{M,1} = a + \delta$) and in the index minimum n becomes; the number of the intersections rises then of $(2n+1)$ to $(2n+3)$ — by scoring the contacts as double counts — and returns immediately to $(2n+1)$, as soon as K exceeds $y_{m,n}$. One has thus generally $2n+1$ roots, which become only with $K = y_{M,1}$ too $(2n+3)$.

In the other case, $\delta > \eta$, the whole amplitude interval $(\delta - \eta) = (y_{M,1} - y_{m,n})$ in that exists the number of the roots of $(2n+1)$ to $(2n+3)$ rises, in order to return to $(2n+1)$, as soon as K exceeds $y_{m,n}$. This value keeps obvious to the end of the interval. Briefly, in this case the things will take to the following process: during K from a to $y_{m,n}$ — this extreme impossible — the number of the roots $2n+1$, it is varied $2n+3$, during K from $y_{m,n}$ to $y_{M,1}$ is varied — extremes included — and returns for the whole remaining section of the period on $2n+1$.

In summary: always accepted that $(2n-1)$ the largest odd whole number is, for which one has

$$(2n-1)\pi \leq y_{M,1} + a$$

then we is gotten, if we have

$$\delta - \eta = y_{M,1} - y_{m,n} = 2y_{M,1} - (2n+1)\pi \geq 0$$

that the number of the roots varies between $(2n+1)$ and $(2n+3)$. In summary it results the two conditions that this case arises, if one has:

$$(2n-1)\pi < y_{M,1} + a < (2n+1)\pi \leq 2y_{M,1}$$

Finally results from the staff analysis, that in order to find the desired number of the roots, if a is given, is sufficient, the ordinate of the 1. maximums with the help of the formula

$$y_{M,1} = a + a \cos \alpha = \arcsin \left[\frac{1}{a} \cdot \sqrt{a^2 - 1} \right]$$

and itself the numbers $(a + y_{M,1})/\pi$ and $2y_{M,1}/\pi$

to form; then, if among them a certain odd whole number $(2n+1)$ is contained, the number of the roots between this and the following odd number varies in the described way; in the opposite case the looked for number varies between the two following, the computed numbers of comprehensive whole numbers.

In the special is in the case, where $(a + y_{M,1})/\pi$ even an odd whole number would be, $(2n-1)$, the number of the roots $(2n+1)$ in the interval $(a, y_{M,1})$ and becomes $(2n-1)$ in the remaining part of the period; in the case, where $2y_{M,1}/\pi$ the odd whole number $(2n-1)$ would be, the looked for number is constant $(2n-1)$ except at one point (with $K = y_{M,1}$), where it becomes $(2n+1)$.

3. If we want to walk now for the real computation of the roots of the equation

$$y = x + a \cos x = K \tag{6}$$

where K is one between a and $a + 2\pi$ contained number, then we can avail ourselves very favorably of the characteristic expressed by the relationship (3), which permits us to limit the investigation on that straight lines parallel after the intersections more certainly to the axis x in each case with the first period¹⁾ of the curve $y(x)$.

The larger clarity because of we accept these roots to be familiar and in increasing order arranged. They are:

$$x_1, x_2, x_3, \dots, x_{2\pi+1}$$

We notice that of them the first three or 1. alone the 1. it belongs to period of the curve ever after $K \leq y_{M,1}$ or $K > y_{M,1}$ is, and that the following belong to two and two to the following periods.

Assumed thus that the two roots x_4 and x_5 of the 2. Belong to period, then we will write

$$x_4 = 2\pi + x_{4,0} \quad x_5 = 2\pi + x_{5,0}$$

The equation (3) means us however that

$$y(x_4) = 2\pi + y(x_{4,0})$$

it is from which one gets, since $y(x_4) = K$:

$$y(x_{4,0}) = K - 2\pi$$

and likewise:

$$y(x_{5,0}) = K - 2\pi$$

The latter means to us that it is sufficient, in order to find the two roots x_4 and x_5 , to draw the equation straight line $y = K - 2\pi$ and to look for their intersections with the first period of the curve. We receive in such a way in the reason the angle values of the phases $x_{4,0}$, $x_{5,0}$, with whose assistance one gets the two looked for values directly, by adding simply the number of 2π in addition.

In perfectly similar way we that it, in order to receive the roots x_6 , x_7 , find, to draw the straight line $y = K - 2 \cdot 2\pi$ the abscissas of the intersections $x_{6,0}$, $x_{7,0}$ been sufficient to look for only with the first period of the curve and to add to them the number of $2 \cdot 2\pi$ etc.

Briefly, we will receive all required roots, by we the intersections of the first period of the curve with the equation degrades:

¹⁾ It is necessary to hold this "first period of the curve", of which we speak here, and the first period of the function sharply apart $y(x)$. First covers indeed all points, whose x falls between 0 and 2π , while latter all points covers, whose y falls between a and $a + 2\pi$, and, their x can have points belonging to the interval $0 < (2n+1)\pi$.

$$\begin{aligned}
 y &= K \\
 y &= K - 2\pi \\
 y &= K - 2 \cdot 2\pi \\
 &\dots\dots\dots \\
 &\dots\dots\dots \\
 y &= K - n \cdot 2\pi
 \end{aligned}
 \tag{7}$$

search and to the in such a way found phases the appropriate multiple ones of 2π add, which occurs in the second member of (7).

After the investigation is so formulated, it is best for the location of the desired phases to design a panel in which the values of y for accepted values of the x for the interval $(0, 2\pi)$ are registered.

Practical the construction of such panels is to be multiplied not with difficulty, because it is sufficient to take a usual trigonometric panel the values cosine - from $30'$ to $30'$ - by the constant of the a and that of the elbow, expressed in parts of the radius to add. Yes, regarding the use of our function for astronomical purposes the production of a certain number of auxiliary tables can be useful, the products of the values of the function $\cos x$ with the sequential natural numbers of 1-9 contain, in such a manner that one can compute with its use easily the product $a \cos x$ for any given value of a and thus the panel

$$y = x + a \cos x$$

for the desired value of a . ¹⁾

4. To select to show turning into to computation of ballistic theory "variable" foreseen light curves prefer we it, concrete example, us not only opportunity gives, as in the concrete one the stated views are used, but also misunderstanding to eliminate, into which I regarding the choice of the upper border, to which the product kb subjected to be would have, so that the phenomenon of the variability was still perceptible, purged. We want to select thus intentionally the case, in which $a = 10$ and therefore kb that value has, which had been regarded in my earlier work due to intuitive views as the border, at which the variability would disappear.

The relationship between observation time and time of departure of the rays of the star:

$$T = k\tau_0 + t + kb\tau_0 \cos \omega t$$

becomes in the accepted case:

$$y = x + 31.41592 \cos x$$

where

$$y = \omega(T - k\tau_0) \quad x = \omega t$$

The first maximum of y has thus instead of at through

$$\alpha = \arcsin(1/10\pi) = 1^\circ 49' 26.7''$$

the given value of x , and the value of the ordinate $y_{M,1}$ is 31.43205.

The numbers $(a + y_{M,1})/\pi$ consequently and $2y_{M,1}/\pi$ have the values 20.0054 and 20.0108, which us, since they both are contained between 19 and 21, it means that the number

of the distinct positions of the star, from which rays arrive at the same time at the observer moves between 19 and 21.

The overall view seen by the observer will thus be formed by the overlay of these 19 or 21 elementary images (the distance between star and observer and the radius of the rotation of path does not permit the separation of these images), and it a brightness will have equal the sum of the brightness of the elemental images.

If one wants to design the light curve to points, then one will have to thus proceed in the following way:

In suitable way a certain number of values of T (observation time), i.e. of y is determined; these are $y_0, y_1, y_2 \dots y_i$.

Accordingly to everyone the same e.g. to y_1 etc. the values x_1 etc. are looked for, which fulfill the equation

$$x + a \cos x = y_i$$

or better the appropriate phases with the help of the equations $x + a \cos x = y_i - m \cdot 2\pi$, which happens with the help of the prepared board and in the way already indicated. Are $x_{i0}, x_{i1}, x_{i2} \dots x_{i,2n+1}$ the 21 (or 19) so found numbers.

One computes then according to each of these phases the absolute value of the derived ones dy/dx , since this, by coinciding with dT/dt , takes a value, which is in reverse proportional to the brightness of the field and can us thus for the measurement the same serve in each point.

If one takes thus the reverse values of the 21 numbers $y(x_{ij})$ and if it adds, then one receives a number, those to the apparent brightness at the moment of the T_i of observed picture is proportional and is given through

$$T(T_i - k\tau_0) = y_i$$

The somewhat toilsome computations were implemented by my assistant Dr. *G. Petrucci*, who I express for it my thanks and my praise.

Below lead I unite examples on:

If one takes the amplitude $a = 31,41592\dots$ as the first value y_0 from y , and imagines one therefore of the first point of contact of the curve with the upper edge of the strip a parallel to the axis x pulled, then one gets the following numbers as phases of the intersections the same with the curve:

	x	J	x	J	x	J
1. Periode	0° 0' 0"	1	3° 38' 0"	1.0526	325° 24' 0"	0.0529
2. »	—	—	38 51	0.0540	310 59	0.0400
3. »	—	—	55 19 50	0.0402	295 45	0.0342
4. »	—	—	68 48	0.0353	284 1	0.0318
5. »	—	—	81 3 30	0.0333	272 46	0.0299
6. »	—	—	92 57 30	0.0331	261 38 45	0.0311
7. »	—	—	104 58 45	0.0339	250 10	0.0328
8. »	—	—	117 43	0.0372	357 51 15	0.0134
9. »	—	—	132 20	0.0450	223 37	0.0442
10. »	—	—	152 12 30	0.0733	204 0	0.0701

¹⁾ It is obvious the fact that, if first leads to three intersections, which last will not be able to give, if the number of the roots is $2n+1$.

After these were found, the absolute values of $1/\dot{y}$ resulted, which is arranged in the attached table in columns the named J (partial light intensities); finally those was computed at the moment $T_0 = k\tau_0 + kb\tau_0$ (which we will take as starting point of the times T) observed total brightness. In the aforementioned case one sees that this sum has the value 2.84. In other words, in this instant $T = T_0$ the star, due to the accepted ballistic reproduction of the light, becomes which appears observers with a 2.84 times so large brightness, when he would have had to show, if the light with constant speed reproduced itself, or if the star did not describe a path.

It is still very important to make attentive on the fact that to the formation of this total brightness the 21 partial images in very different measure contribute.

In measure outweighing far in addition two first of the three carry to the 1 with period of pictures belonging to; the first with the value 1, the other one with the value 1.0526, while the remaining 19 images supply in addition among themselves little different contributions of the size of some hundredths, so that, while two first for itself supply a contribution of 2.05 alone for the brightness of the overall view, the remaining 19 does not reach collected a contribution of 0.8!!

In the same way the brightness was computed, which is offered to the observer at the times indicated in the first column of the following table. These times are certain and in fractions of the period T_0 (i.e. with acceptance of $\tau_0 = 1$) expressed, on the basis of the instant indicated above.

	J		J
0.000000	2.8411	0.487500	2.1113
0.004838	Max.	0.49490	Max.
0.013888	1.0035	0.51388	1.48744
0.027776	0.7845	0.52776	1.28407
0.035552	0.7842	0.666717	0.97692
0.083333	0.7802	0.83333	0.98390
0.250000	0.7656	0.97222	1.28424
0.472222	0.7794	0.98610	1.48110
0.486060	0.7818	1.00000	2.84110

The described procedure encounters a serious difficulty, if it is applied to the evaluation the maxima of the light curve appropriate brightness (i.e. if as values of K the ordinates of the 1. maximum and the last left minimum lying from $2a$ to be selected).

Indeed becomes, there then under the x_i the abscissa of the maximum (or the minimum) figures, one the y zero, which brings with itself that one would have to attribute an infinite brightness to the appropriate elementary picture. It became in this. To case appear many more appropriate, the brightness of this "frame" by the value of the relationship taken outside of the limit to derive $\Delta y/\Delta x$. In addition, the numbers, which will receive in this way, depend strongly on the special value, which one attaches (arbitrary) Δx , so that it in the absence of

a criterion for the choice of Δx is not possible to rely on one of those, which one can compute.

One must thus, in order to derive the values of the maxima, differently proceed.

It is easily evident that the best criterion is the following: to draw graphically the curve due to the points already found (i.e. it without the maximum pull) and latter on it specify, by one consider, that the area between the curve, the x axis and the outermost ordinate alike be must the rectangle, which the period and the effective brightness 1 the star to side have. This, because the two areas respectively the total quantities of light represent, which must arrive in one period at the observer with the ballistic and with the usual hypothesis and which from evident energetic requirements (1st principle) the same to be to have.

With application of this criterion in the questionable concrete case we received the numbers of 4.06 and 4.92 as values for the two looked for maxima.

Briefly, despite the constant of a of given relatively large value (10) the ballistic hypothesis leads us there, a light curve to foresee, which does not only exhibit the phenomenon of the variability still very clearly, but the characteristics possesses, which the mentioned phenomenon in many observed cases exhibits.

The foreseen light curve shows indeed two very pronounced, extremely abrupt and very short continuing maxima in the phase of the ascent, which by two very flat minima are separate, in which the brightness long time through remains nearly constant. The amplitude of the change of brightness in the computed example is from the order of magnitude of two stages in the scale of the star sizes, and their phases are practically as follows distributed:

Two long intervals, in those the brightness is noticeably decreased with the values, which it has in the two minima (0.78 and/or 0.92), which cover in the whole 60/100 of the period; the two phases of the acceptance, which cover 20/100, those the slow increase on 15/100 and the two maxima, which develop nearly completely into the remaining 5/100 of the period, although the largest part of the ascent to the maximum and the descent carries out itself in an extremely short time.

The same results are found, if many larger values are attached to the constant of a , as I could determine by an orienting computation for the case $a = 200\pi$. Everything that is registered in the light curve, is a smaller duration of the maxima and a corresponding increase of their brightness. We can thus close that no upper border exists for the constant a (and thus for kb), beyond which the phenomenon of the variability any longer do not show up. The conflict between the consequences from the ballistic theory and the observations stated by *Bernheimer* and *Salet* is thus not substantial and is limited to a divergence the form of the light curve, which can be easily switched off.

The characteristics, with which the light change foreseen by the ballistic theory for these large values of a arises, cover themselves indeed not with those for the variable one of the Algol type, follow rather much

close that, those with the variable ones of the 3. Group, the Class IV of the *Pickering* classification, i.e. group the called "type of cluster" (which the most numerous from all is) and the long-periodic variable ones of the Class IIB the same classification are observed. ¹⁾

From a paper on *S. I. Bailey* ²⁾ we bring 2, 3, 4 some light curves of stars belonging to that type and also the following table in the illustrations,

in that the most out standing phases observed with the phenomenon of the variability with some stars in the star cluster M_3 are schematically represented.

Duration of	Maximums	0/100
" "	Minimums	40/100
" "	Decline	50/100
" "	Decline	10/100
		100/100

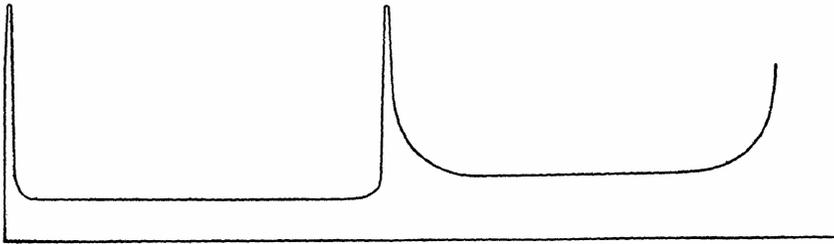


Fig. 2

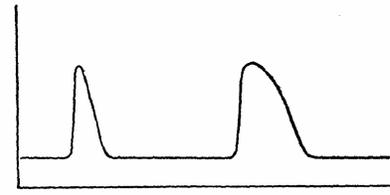


Fig. 3

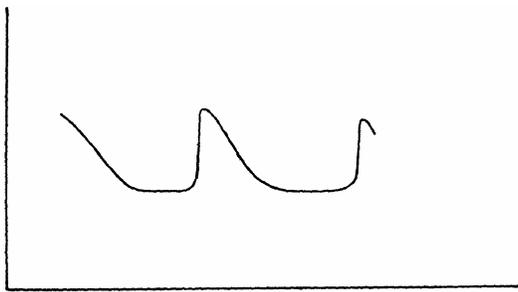


Fig. 4

This table and those light curves show clearly the parallelism of the behavior between these cases of the observation and the theoretically derived curve.

Similar characteristics are possessed by *SS Cygni* (Fig. 3) (the most typical and strangest variable one that

Class IIB from *Pickering*) observed light curve, which exhibits likewise two long intervals of the light being constant (minimum), two nearly presently/immediately flashing maxima etc. Also still far must be considered that the ballistic theory had good reserves, in order from everyone the variable one of this important and mysterious group, which until today to each attempt at explanation escaped, to be able to explain offered features more near, by availing themselves in addition: 1. the choice of the constant a made in the most suitable way; 2. the ellipticity of the path (which we assumed for the sake of simplicity here as circular); 3. the same orientation regarding the face line etc. In this way the different duration of the two maxima appears to explain the different speed of the two descents etc. as possible.

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¹⁾ Comparisons *K. Schiller*, introduction to the study of the variable stars, A. Barth, Leipzig, 1923.

²⁾ *ApJ* 10.260 (1899).